

SOME FIXED POINT RESULTS IN ORDERED PARTIAL METRIC SPACES

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ABSTRACT. In this paper, we establish some fixed point theorems in ordered partial metric spaces. An example is given to illustrate our obtained results.

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1. INTRODUCTION AND PRELIMINARIES

When fixed point problems in partially ordered metric spaces are concerned, first results were obtained by Ran and Reurings [17], and then by Nieto and López [11]. The following fixed point theorem was proved in these papers.

Theorem 1.1. [11, 17] *Let (X, \leq_X) be a partially ordered set and let d be a metric on X such that (X, d) is a complete metric space. Let $f : X \rightarrow X$ be a non-decreasing map with respect to \leq_X . Suppose that the following conditions hold:*

- (i) $\exists 0 \leq c < 1$, $d(fx, fy) \leq cd(x, y)$ for any $y \leq_X x$
- (ii) $\exists x_0 \in X$ such that $x_0 \leq_X fx_0$
- (iii) f is continuous in (X, d) , or
- (iii') if a non-decreasing sequence $\{x_n\}$ converges to $x \in X$, then $x_n \leq_X x$ for all n .

Then f has a fixed point $u \in X$.

Results on weakly contractive mappings in such spaces were obtained by Harjani and Sadarangani in [8]. An extension of the previous result is the following

Theorem 1.2. [8] *Let (X, \leq) be a partially ordered set and suppose that there exists a metric d in X such that (X, d) is a complete metric space. Let $f : X \rightarrow X$ be a non-decreasing mapping with respect to \leq_X such that*

$$d(fx, fy) \leq d(x, y) - \psi(d(x, y)),$$

for $y \leq_X x$, where $\psi : [0, +\infty[\rightarrow [0, +\infty[$ is a continuous and non-decreasing function such that it is positive in $]0, +\infty[$, $\psi(0) = 0$ and $\lim_{t \rightarrow +\infty} \psi(t) = +\infty$. Assume that

- (i) f is continuous in (X, d) , or
 - (ii) if a non-decreasing sequence $\{x_n\}$ converges to $x \in X$, then $x_n \leq_X x$ for all n .
- If there exists $x_0 \in X$ with $x_0 \leq_X fx_0$, then f has a fixed point.

Many other results on the existence of fixed points or common fixed points in ordered metric spaces were given, we can cite for example [1, 3, 5, 6, 7, 9, 12, 15, 21, 23] and the references therein.

In this paper we extend the results of Harjani and Sadarangani [8] to the case of partial metric spaces. An example is considered to illustrate our obtained results.

First, we start with some preliminaries on partial metric spaces. For more details, we refer the reader to [2, 4, 10, 13, 14, 16, 18, 19, 20, 22, 23].

Definition 1.3. Let X be a nonempty set. A partial metric on X is a function $p : X \times X \rightarrow R_+$ such that for all $x, y, z \in X$:

$$(p1) \quad x = y \iff p(x, x) = p(x, y) = p(y, y),$$

$$(p2) \quad p(x, x) \leq p(x, y),$$

$$(p3) \quad p(x, y) = p(y, x),$$

$$(p4) \quad p(x, y) \leq p(x, z) + p(z, y) - p(z, z).$$

A partial metric space is a pair (X, p) such that X is a nonempty set and p is a partial metric on X .

If p is a partial metric on X , then the function $p^s : X \times X \rightarrow R_+$ given by

$$p^s(x, y) = 2p(x, y) - p(x, x) - p(y, y),$$

is a metric on X .

Definition 1.4. Let (X, p) be a partial metric space. Then:

(i) a sequence $\{x_n\}$ in a partial metric space (X, p) converges to a point $x \in X$ if and only if $p(x, x) = \lim_{n \rightarrow +\infty} p(x, x_n)$;

(ii) a sequence $\{x_n\}$ in a partial metric space (X, p) converges properly to a point $x \in X$ if and only if $p(x, x) = \lim_{n \rightarrow +\infty} p(x_n, x_n) = \lim_{n \rightarrow +\infty} p(x, x_n)$, if and only if $\lim_{n \rightarrow +\infty} p^s(x, x_n) = 0$;

(iii) A sequence $\{x_n\}$ in a partial metric space (X, p) is called a Cauchy sequence if there exists (and is finite) $\lim_{n, m \rightarrow +\infty} p(x_n, x_m)$;

(iv) A partial metric space (X, p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges to a point $x \in X$, that is $p(x, x) = \lim_{n, m \rightarrow +\infty} p(x_n, x_m)$.

Lemma 1.5. Let (X, p) be a partial metric space.

(a) $\{x_n\}$ is a Cauchy sequence in (X, p) if and only if it is a Cauchy sequence in the metric space (X, p^s) ;

(b) A partial metric space (X, p) is complete if and only if the metric space (X, p^s) is complete. Furthermore, $\lim_{n \rightarrow +\infty} p^s(x_n, x) = 0$ if and only if

$$p(x, x) = \lim_{n \rightarrow +\infty} p(x_n, x) = \lim_{n, m \rightarrow +\infty} p(x_n, x_m).$$

Definition 1.6. Suppose that (X_1, p_1) and (X_2, p_2) are partial metrics. Denote $\tau(p_1)$ and $\tau(p_2)$ their respective topologies. We say $T : (X_1, p_1) \rightarrow (X_2, p_2)$ is continuous if both $T : (X_1, \tau(p_1)) \rightarrow (X_2, \tau(p_2))$ and $T : (X_1, \tau(p_1^s)) \rightarrow (X_2, \tau(p_2^s))$ are continuous.

Proposition 1.7. Let (X, p) be a partial metric space, partially ordered and $T : X \rightarrow X$ be a given mapping. We say that T is continuous in $x_0 \in X$ if for every sequence $\{x_n\}$ is X , we have

(a) x_n converges to x_0 in (X, p) implies Tx_n converges to Tx_0 in (X, p) .

(b) x_n converges properly to x_0 in (X, p) implies Tx_n converges properly to Tx_0 in (X, p) .

If T is continuous on each point $x_0 \in X$, then we say that T is continuous on X .

Definition 1.8. If (X, \leq_X) is a partially ordered set and $f : X \rightarrow X$, we say that f is monotone nondecreasing if $x, y \in X$, $x \leq_X y$ implies $fx \leq_X fy$.

2. MAIN RESULTS

Our first result is the following theorem

Theorem 2.1. Let (X, \leq_X) be a partially ordered set and let p be a partial metric on X such that (X, p) is complete. Let $f : X \rightarrow X$ be a non-decreasing map with respect to \leq_X . Suppose that the following conditions hold: for $y \leq x$, we have

(i)

$$(2.1) \quad p(fx, fy) \leq p(x, y) - \psi(p(x, y)),$$

where $\psi : [0, +\infty[\rightarrow [0, +\infty[$ is a continuous and non-decreasing function such that it is positive in $]0, +\infty[$, $\psi(0) = 0$ and $\lim_{t \rightarrow +\infty} \psi(t) = +\infty$;

(ii) $\exists x_0 \in X$ such that $x_0 \leq_X fx_0$;

(iii) f is continuous in (X, p) , or;

(iii') if a non-decreasing sequence $\{x_n\}$ converges to $x \in X$, then $x_n \leq_X x$ for all n .

Then f has a fixed point $u \in X$. Moreover, $p(u, u) = 0$.

Proof. Let $x_0 \in X$ be such that $x_0 \leq_X fx_0$. As f is monotone non-decreasing, then

$$x_0 \leq_X fx_0 \leq_X f^2x_0 \leq_X f^3x_0 \leq_X \dots \leq_X f^nx_0 \leq_X f^{n+1}x_0 \leq_X \dots$$

Put $x_n = f^n x_0$, then for any $n \in \mathbb{N}^*$, we have $x_{n-1} \leq_X x_n$. Then for each integer $n \geq 1$, from (2.1) and, as the elements x_n and x_{n-1} are comparable, we get

$$(2.2) \quad p(x_{n+1}, x_n) = p(fx_n, fx_{n-1}) \leq p(x_n, x_{n-1}) - \psi(p(x_n, x_{n-1})),$$

If there exists $n_1 \in \mathbb{N}^*$ such that $p(x_{n_1}, x_{n_1-1}) = 0$, then $x_{n_1-1} = x_{n_1} = fx_{n_1-1}$ and x_{n_1-1} is a fixed point of f and the proof is finished. In other case, suppose that $p(x_{n+1}, x_n) \neq 0$ for all $n \in \mathbb{N}$. Then, using an assumption on ψ in (2.2) we have for $n \in \mathbb{N}^*$

$$p(x_{n+1}, x_n) \leq p(x_n, x_{n-1}) - \psi(p(x_n, x_{n-1})) < p(x_n, x_{n-1}).$$

Put $\rho_n = p(x_{n+1}, x_n)$, then we have

$$(2.3) \quad \rho_n \leq \rho_{n-1} - \psi(\rho_{n-1}) < \rho_{n-1}.$$

Therefore $\{\rho_n\}$ is a nonnegative non-increasing sequence and hence possesses a limit ρ^* . From (2.3), taking limit when $n \rightarrow +\infty$, we get

$$\rho^* \leq \rho^* - \psi(\rho^*),$$

and, consequently, $\psi(\rho^*) = 0$. By our assumptions on ψ , we conclude $\rho^* = 0$, that is,

$$(2.4) \quad \lim_{n \rightarrow +\infty} p(x_n, x_{n+1}) = 0.$$

In what follows we shall show that $\{x_n\}$ is a Cauchy sequence in the partial metric space (X, p) . Fix $\varepsilon > 0$, as $\rho_n = p(x_{n+1}, x_n) \rightarrow 0$, there exists $n_0 \in \mathbb{N}$ such that

$$(2.5) \quad p(x_{n_0+1}, x_{n_0}) \leq \min\left\{\frac{\varepsilon}{2}, \psi\left(\frac{\varepsilon}{2}\right)\right\}.$$

We claim that if $z \in X$ verifies $p(z, x_{n_0}) \leq \varepsilon$ and $x_{n_0} \leq_X z$, we get $p(fz, x_{n_0}) \leq \varepsilon$. Indeed, to do this we distinguish two cases :

Case 1. $p(z, x_{n_0}) \leq \frac{\varepsilon}{2}$. In this case, as z and x_{n_0} are comparable, we have

$$\begin{aligned} p(fz, x_{n_0}) &\leq p(fz, fx_{n_0}) + p(fx_{n_0}, x_{n_0}) \\ &= p(fz, fx_{n_0}) + p(x_{n_0+1}, x_{n_0}) \\ &\leq p(z, x_{n_0}) - \psi(p(z, x_{n_0})) + p(x_{n_0+1}, x_{n_0}) \\ &\leq p(z, x_{n_0}) + p(x_{n_0+1}, x_{n_0}) \\ &\leq \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon. \end{aligned}$$

Case 2. $\frac{\varepsilon}{2} \leq p(z, x_{n_0}) \leq \varepsilon$. In this case, as ψ is a non-decreasing function, $\psi(\frac{\varepsilon}{2}) \leq \psi(p(z, x_{n_0}))$. Therefore, from (2.5) we have

$$\begin{aligned} p(fz, x_{n_0}) &\leq p(fz, fx_{n_0}) + p(fx_{n_0}, x_{n_0}) \\ &= p(fz, fx_{n_0}) + p(x_{n_0+1}, x_{n_0}) \\ &\leq p(z, x_{n_0}) - \psi(p(z, x_{n_0})) + p(x_{n_0+1}, x_{n_0}) \\ &\leq p(z, x_{n_0}) - \psi(\frac{\varepsilon}{2}) + p(x_{n_0+1}, x_{n_0}) \\ &\leq p(z, x_{n_0}) - \psi(\frac{\varepsilon}{2}) + \psi(\frac{\varepsilon}{2}) \\ &\leq p(z, x_{n_0}) \leq \varepsilon. \end{aligned}$$

This proves the claim. As x_{n_0+1} verifies $p(x_{n_0+1}, x_{n_0}) \leq \varepsilon$ and $x_{n_0} \leq_X x_{n_0+1}$, the claim gives us that $x_{n_0+2} = fx_{n_0+1}$ verifies $p(x_{n_0+2}, x_{n_0}) \leq \varepsilon$. We repeat this process to get

$$p(x_n, x_{n_0}) \leq \varepsilon \quad \text{for any } n \geq n_0.$$

It follows that

$$p(x_n, x_m) \leq p(x_n, x_{n_0}) + p(x_{n_0}, x_m) \leq \varepsilon + \varepsilon = 2\varepsilon \quad \text{for any } n, m \geq n_0.$$

Since ε is arbitrary, then $\lim_{n, m \rightarrow +\infty} p(x_n, x_m) = 0$. Thus, $\{x_n\}$ is a Cauchy sequence in (X, p) , so by Lemma 1.5, $\{x_n\}$ is a Cauchy sequence in the metric space (X, p^s) . Since (X, p) is complete, hence (X, p^s) is complete, so there exists $u \in X$ such that

$$(2.6) \quad \lim_{n \rightarrow +\infty} p^s(x_n, u) = 0.$$

Thus, by Lemma 1.5, from the condition (p2) and (2.4), we get

$$(2.7) \quad p(u, u) = \lim_{n \rightarrow +\infty} p(x_n, u) = \lim_{n \rightarrow +\infty} p(x_n, x_n) = 0.$$

We prove now that $fu = u$. We shall distinguish the cases (iii) and (iii') of the Theorem 2.1.

Case 1. Suppose that the mapping f is continuous. In particular, thanks to condition (b) in proposition 1.7, we have fx_n converges properly to fu in (X, p) , that is $p^s(fx_n, fu) \rightarrow 0$, since $p^s(x_n, u) \rightarrow 0$, i.e., $\{x_n\}$ converges properly to u in (X, p) . Hence we have $\{fx_n\}$ converges to fu in (X, p^s) . On the other hand, $\{fx_n = x_{n+1}\}$ converges to u in (X, p^s) because of (2.6). By uniqueness of the limit in metric space (X, p^s) , we deduce that $fu = u$.

Case 2. Suppose now that the condition (iii') of the theorem holds. The sequence

$\{x_n\}$ is non-decreasing with respect to \leq_X , and it follows that $x_n \leq_X u$. Take $x = x_n$ and $y = u$ (which are comparable) in (2.1) to obtain that

$$(2.8) \quad p(u, fu) \leq p(u, x_{n+1}) + p(x_{n+1}, fu) \leq p(u, x_{n+1}) + p(u, x_n) - \psi(p(u, x_n)).$$

Letting $n \rightarrow +\infty$ in (2.8) we find using (2.7) and the properties of ψ that

$$p(fu, u) \leq 0 - \psi(0) = 0,$$

hence $p(fu, u) = 0$, so $fu = u$. This completes the proof of Theorem 2.1. \square

Corollary 2.2. *Let (X, \leq_X) be a partially ordered set and let p be a partial metric on X such that (X, p) is complete. Let $f : X \rightarrow X$ be a non-decreasing map with respect to \leq_X . Suppose that the following conditions hold:*

(i) $\exists 0 \leq c < 1$ such that

$$(2.9) \quad p(fx, fy) \leq cp(x, y) \quad \text{for any } y \leq_X x.$$

(ii) $\exists x_0 \in X$ such that $x_0 \leq_X fx_0$;

(iii) f is continuous in (X, p) , or;

(iii') if a non-decreasing sequence $\{x_n\}$ converges to $x \in X$, then $x_n \leq_X x$ for all n .

Then f has a fixed point $u \in X$. Moreover, $p(u, u) = 0$.

Proof. We take $\psi(t) = (1 - c)t$ in Theorem 2.1.

Next theorem gives a sufficient condition for the uniqueness of the fixed point.

Theorem 2.3. *Let all the conditions of Theorem 2.1 be fulfilled and let the following condition be satisfied: for arbitrary two points $x, y \in X$ there exists $z \in X$ which is comparable with both x and y . Then the fixed point of f is unique.*

Proof. Let u and v be two fixed points of f , i.e., $fu = u$ and $fv = v$. Consider the following two cases:

Case 1. u and v are comparable. Then we can apply condition (2.1) and obtain that

$$p(u, v) = p(fu, fv) \leq p(u, v) - \psi(p(u, v)),$$

hence $\psi(p(u, v)) \leq 0$, i.e, $p(u, v) = 0$, so $u = v$, that is the uniqueness of the fixed point of f .

Case 2. Suppose now that u and v are not comparable. Choose an element $w \in X$ comparable with both of them. Then also, $u = f^n u$ is comparable with $f^n w$ for each n (since f is non-decreasing). Applying (2.1), one obtains for $n \in \mathbb{N}^*$ that

$$\begin{aligned} p(u, f^n w) &= p(f f^{n-1} u, f f^{n-1} w) \leq p(f^{n-1} u, f^{n-1} w) - \psi(p(f^{n-1} u, f^{n-1} w)) \\ &\leq p(f^{n-1} u, f^{n-1} w) = p(u, f^{n-1} w). \end{aligned}$$

It follows that the sequence $\{p(u, f^n w)\}$ is non-increasing and it has a limit $l \geq 0$. Assuming that $l > 0$ and passing to the limit in the relation

$$p(u, f^n w) \leq p(u, f^{n-1} w) - \psi(p(u, f^{n-1} w)),$$

one obtains that $l = 0$, a contradiction. In the same way it can be deduced that $p(v, f^n w) \rightarrow 0$ as $n \rightarrow +\infty$. Now, passing to the limit in $p(u, v) \leq p(u, f^n w) + p(f^n w, v)$, it follows that $p(u, v) = 0$, so $u = v$, and the uniqueness of the fixed point is proved. \square

Example 2.4. Let $X = [0, +\infty[$ endowed with the usual partial metric p defined by $p : X \times X \rightarrow [0, +\infty[$ with $p(x, y) = \max\{x, y\}$. We give the partial order on X by

$$x \leq_X y \iff p(x, x) = p(x, y) \iff x = \max\{x, y\} \iff y \leq x.$$

It is clear that (X, \leq_X) is totally ordered. The partial metric space (X, p) is complete because (X, p^s) is complete. Indeed, for any $x, y \in X$,

$$\begin{aligned} p^s(x, y) &= 2p(x, y) - p(x, x) - p(y, y) = 2\max\{x, y\} - (x + y) \\ &= |x - y|, \end{aligned}$$

Thus, $(X, p^s) = ([0, +\infty[, |\cdot|)$ is the usual metric space, which is complete. Again, we define

$$f(t) = \frac{1}{2}t, \quad \text{if } t \geq 0.$$

The function f is continuous on (X, p) . Indeed, let $\{x_n\}$ be a sequence converging to x in (X, p) , then

$$\lim_{n \rightarrow +\infty} \max\{x_n, x\} = \lim_{n \rightarrow +\infty} p(x_n, x) = p(x, x) = x$$

hence by definition of f , we have

(2.10)

$$\lim_{n \rightarrow +\infty} p(fx_n, fx) = \lim_{n \rightarrow +\infty} \max\{fx_n, fx\} = \lim_{n \rightarrow +\infty} \frac{1}{2} \max\{x_n, x\} = \frac{1}{2}x = p(fx, fx),$$

that is $\{f(x_n)\}$ converges to $f(x)$ in (X, p) . On the other hand, if $\{x_n\}$ converges properly to x in X , hence

$$\lim_{n \rightarrow +\infty} p^s(x_n, x) = 0.$$

Thus, by definition of p^s and f , one can find

(2.11)

$$\lim_{n \rightarrow +\infty} p^s(fx_n, fx) = 0.$$

Both convergences (2.10)-(2.11) yield that f is continuous on (X, p) . Any $x, y \in X$ are comparable, so for example we take $x \leq_X y$, and then $p(x, x) = p(x, y)$, so $y \leq x$. Since $f(y) \leq f(x)$, so $f(x) \leq_X f(y)$, giving that f is monotone non-decreasing with respect to \leq_X . In particular, for any $x \leq_X y$, we have

$$(2.12) \quad p(x, y) = x, \quad p(fx, fy) = f(x) = \frac{1}{2}x.$$

Let us take $\psi : [0, +\infty[\rightarrow [0, +\infty[$ such that $\psi(t) = \frac{1}{4}t$. We have for any $x \in X$, $\frac{1}{2}x \leq x - \frac{1}{4}x$. Consequently, we get for any $x \leq_X y$, thanks to this and (2.12)

$$p(fx, fy) \leq p(x, y) - \psi(p(x, y)),$$

that is (2.1) holds. All the hypotheses of Theorem 2.1 are satisfied, so f has a unique fixed point in X , which is $u = 0$.

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